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Professional Correspondence

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4 February 1997

Hiroshi Akima
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NTIA / ITS
325 Broadway
Boulder, CO 80303

Dear Dr. Akima:

It has been a long time since we have corresponded about interpolation methods. Since then there have been a lot of changes in the Computing Directorate here at LLNL and I haven't had a chance to keep up with the literature until recently.

This letter is prompted by your article entitled, "Note on Local Methods of Univariate Interpolation," which appeared in the April 1996 issue of the *SIGNUM Newsletter*. In that article you compared various univariate methods and conclude that your Algorithm 697 is probably the best. Because I had let my subscription to *ACM Trans. Math. Software* lapse for a few years, I was not familiar with Algorithm 697, so the first thing I did was go to the library and get a copy of your article.

I notice that our routine PCHIC, which you refer to as "the F-C-B method", is mentioned briefly in the TOMS article, but does not appear in any of the comparisons in either paper. (PCHIC can be obtained via Netlib, http://www.netlib.org/ or http://netlib.bell-labs.com/.) The purpose of these notes is to provide information about PCHIC comparable to what you gave in the newsletter article.

Since neither you nor Ellis and McLain tell precisely how the "average deviation from the original function" is computed, I first implemented the Ellis-McLain algorithm (Algorithm 514) and attempted to reproduce the numbers given for the five analytic functions in their paper. I found that using 1000 uniformly spaced subintervals (1001 uniformly spaced evaluation points, including the endpoints) gave comparable results. I also did the computations for Algorithm 697 (cubic method only) and two PCHIC options, one using the default endpoints (PCHIC0) and the other matching the endpoint derivatives of the exact functions (PCHIC1). All PCHIC runs used `SWITCH=-1`. The results (x 10^6) are as follows:

	Method			
Function	Alg. 514	Alg. 697	PCHIC0	PCHIC1
x^3	0	0	15915	13562
x^4	6413	5591	77824	51589
$\exp(-x^2/2)$	76	122	843	840
$\tanh x$	139	132	540	527
$\sin x$	179	135	1690	1228

I presume that the differences between some of these results and those in the newsletter article are due to the fact that I did not use exactly the same evaluation points as you.

Since there is second derivative discontinuity only at the interior data points, I took the phrase "average discontinuity of the second derivative at data points" to be the sum of the absolute values of the second derivative jumps (which can be computed exactly) at the 16 interior points, divided by 16. The results (x 10^4) are as follows:

	Method			
Function	Alg. 514	Alg. 697	PCHIC0	PCHIC1
x^3	0	0	73587	73649
x^4	5766	32771	255814	244763
$\exp(-x^2/2)$	268	833	8620	8506
$\tanh x$	469	931	5494	5464
sin x	198	295	10485	10800

I do not have an explanation for the differences between the numbers for Alg. 514 and 697 and those in your article. Nonetheless, these seem to bear out your claims for Alg. 697 when interpolating data from analytic functions, although they show PCHIC to be comparable to Alg. 433.

I am not sure why you included graphical results for any of these examples in you article, since all methods give comparable-appearing curves. In fact, when I superimposed the curves for Alg. 697 and those for PCHIC they appeared to be identical.

I also computed the PCHIC0 results for all of the sample data sets in your Algorithm 697 TOMS article, and enclose plots comparing them to Alg. 514 and Alg. 697. In order to investigate strange behavior of PCHIC near x=1, I have also included plots for two modifications of the sin(πx) data: modification A includes data for x=0.75 and x=1.25; mod. B instead includes data for x=0.60 and x=1.40.

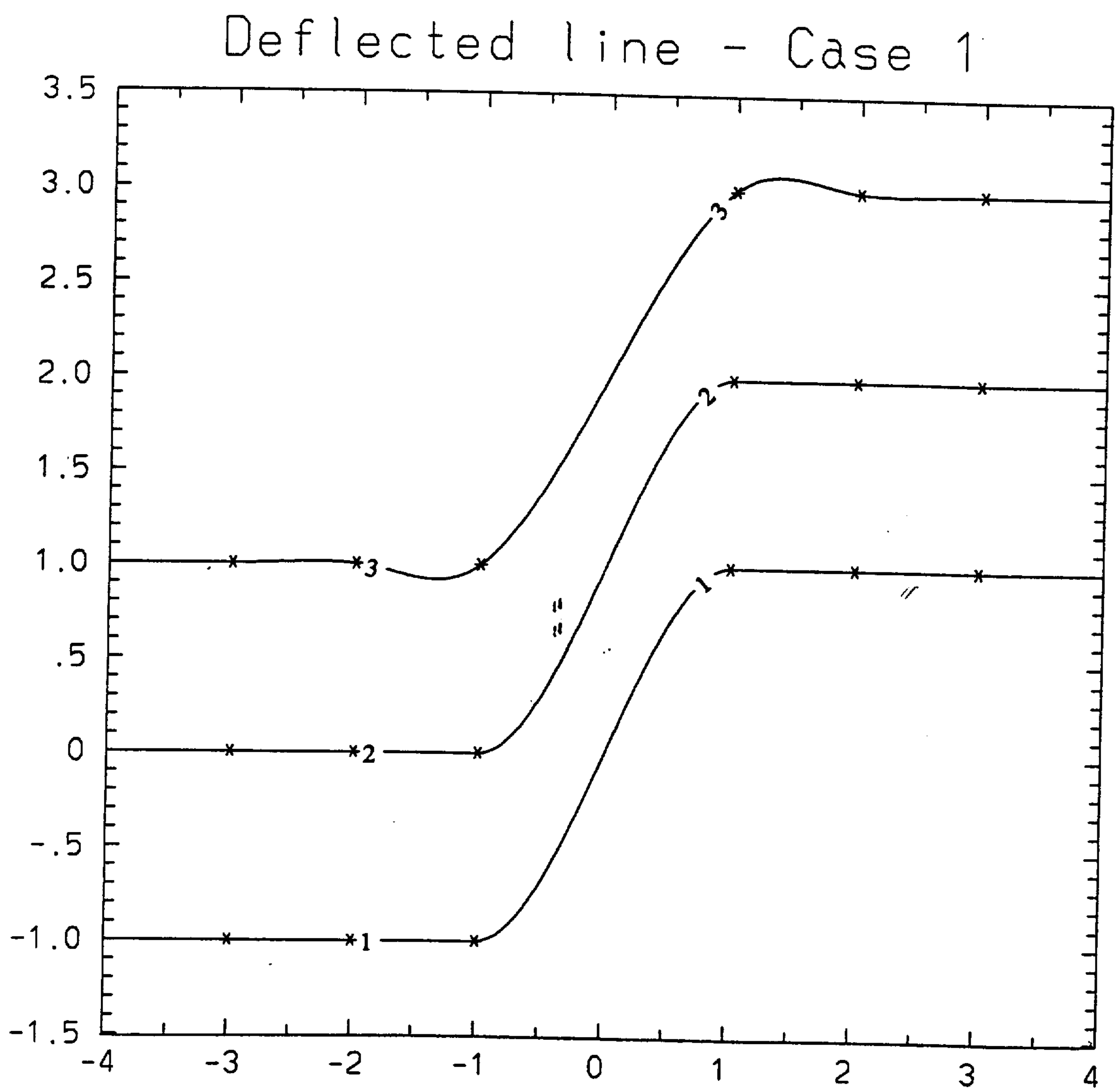
I conclude from these results that Algorithm 697 is superior to PCHIC on sparse analytic data, PCHIC gives more "visually pleasing" results on all the Akima data sets, and the two are comparable on "reasonable" data sets, where sufficient data are given to represent the shape of the underlying phenomenon. The appearance of some of these curves indicates that some work is needed for PCHIC to produce visually pleasing results on nonmonotone data.

Sincerely,

Fred N. Fritsch

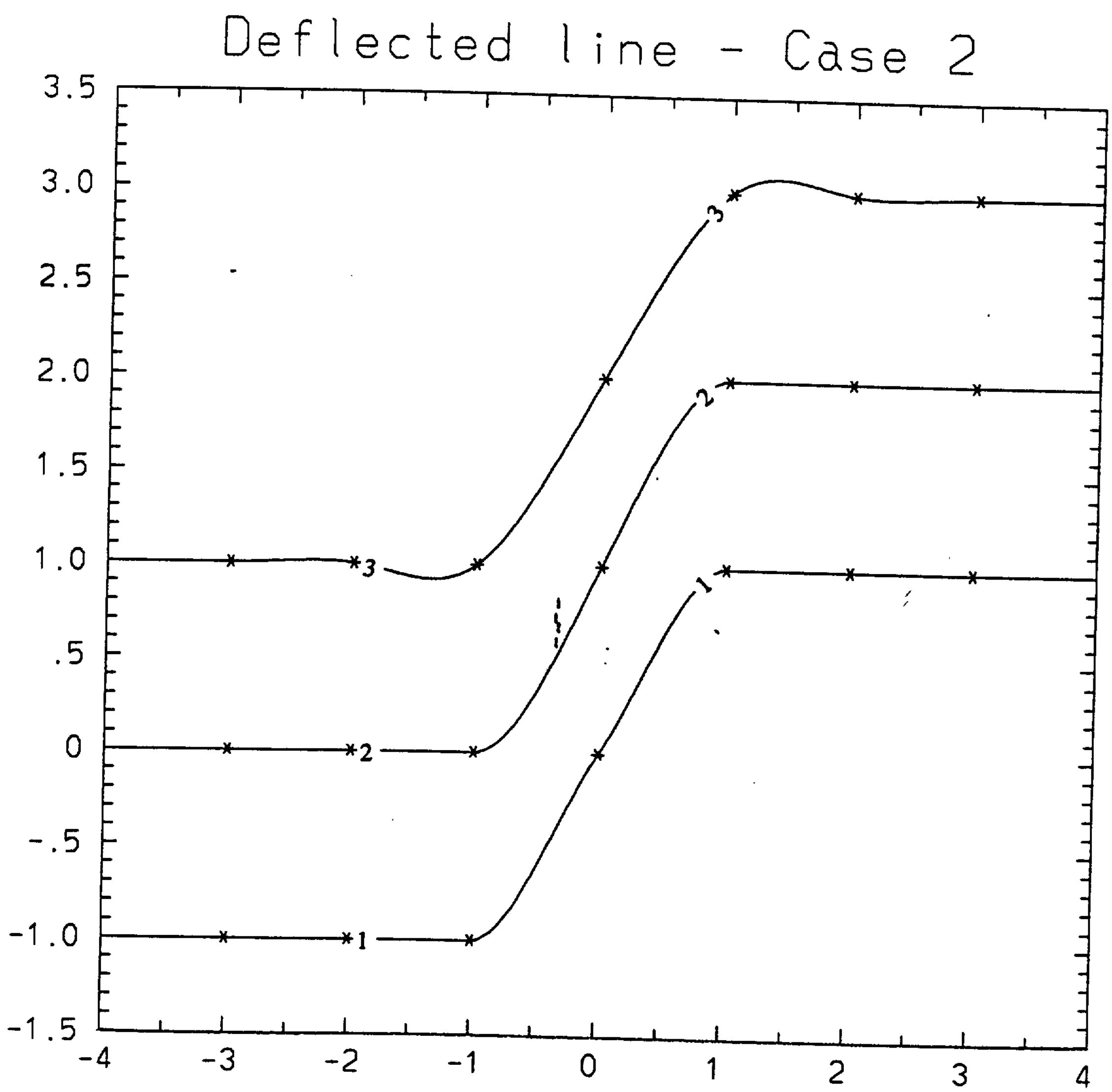
Fred N. Fritsch (L-477)
Computer Applications Organization
LCPD / ICF Group

Fig. 1



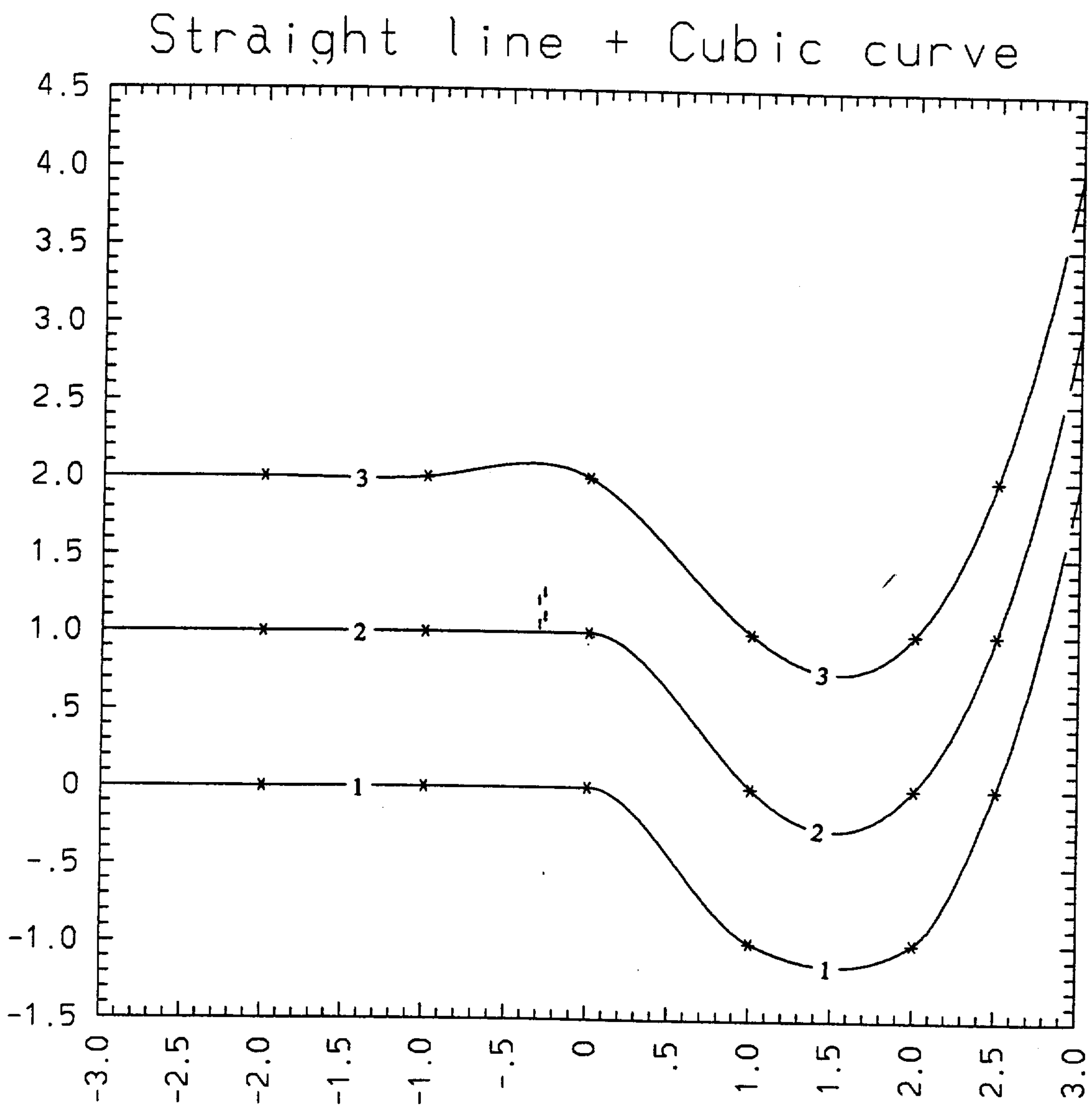
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 2



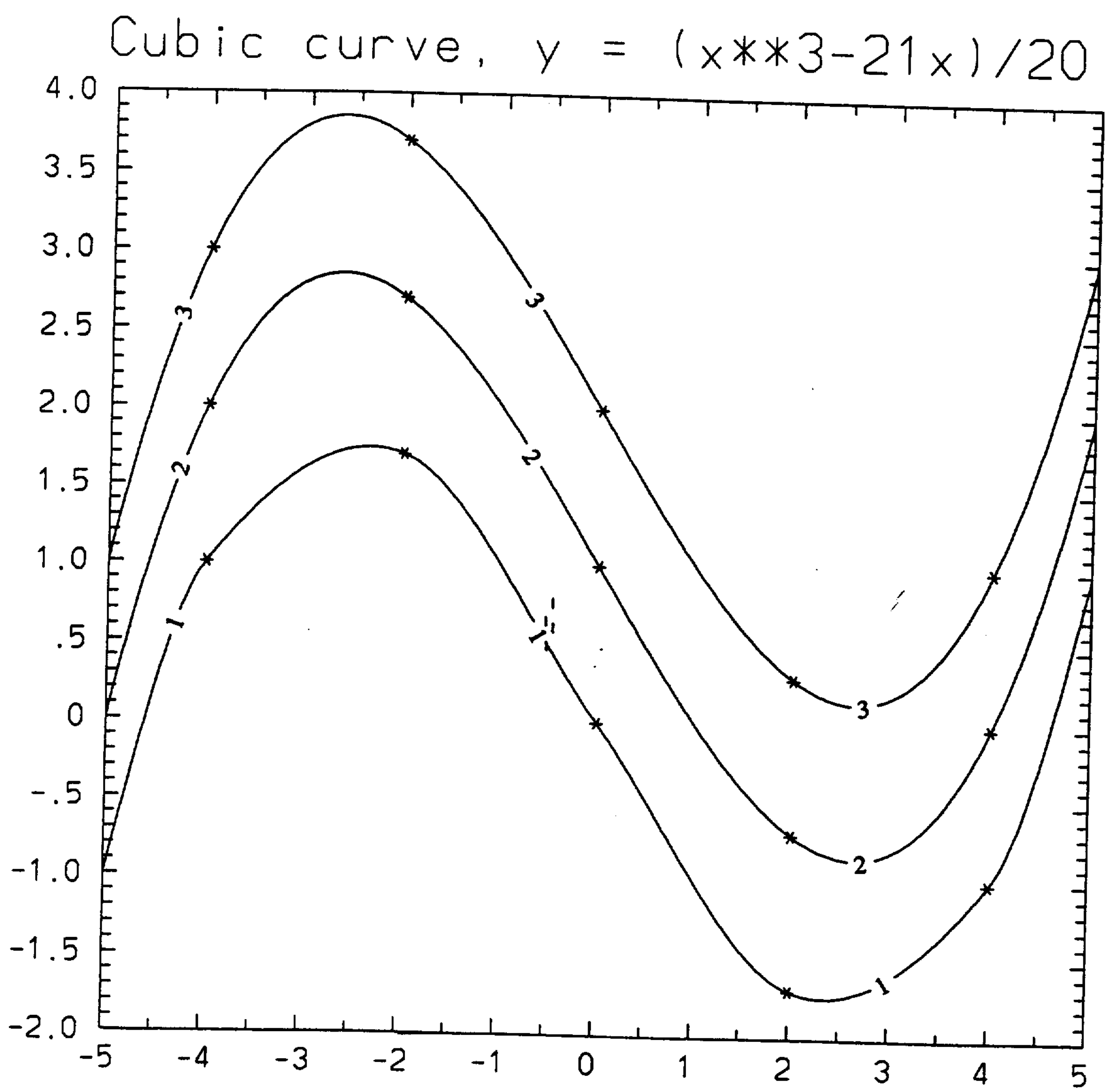
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 3



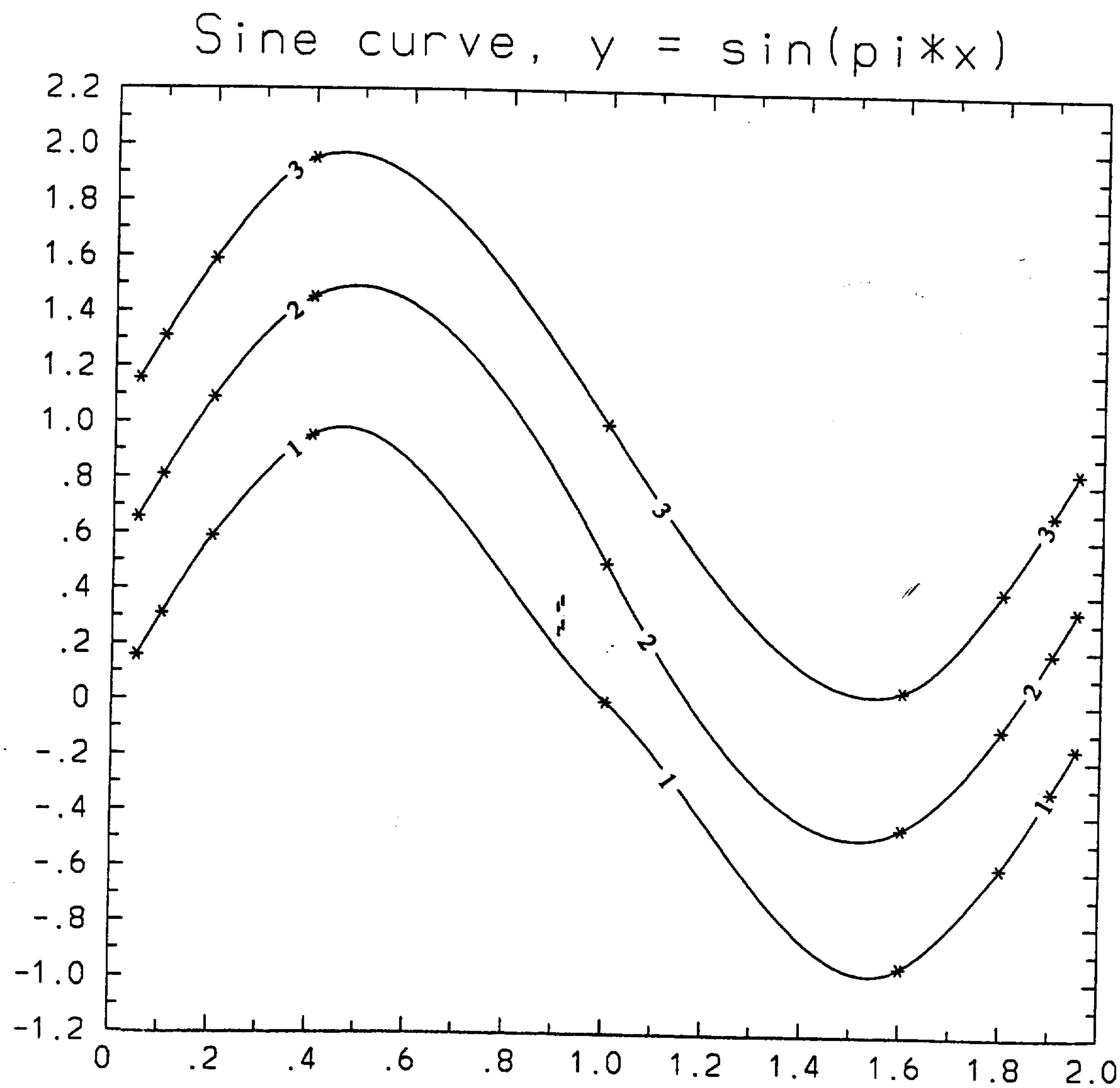
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 4



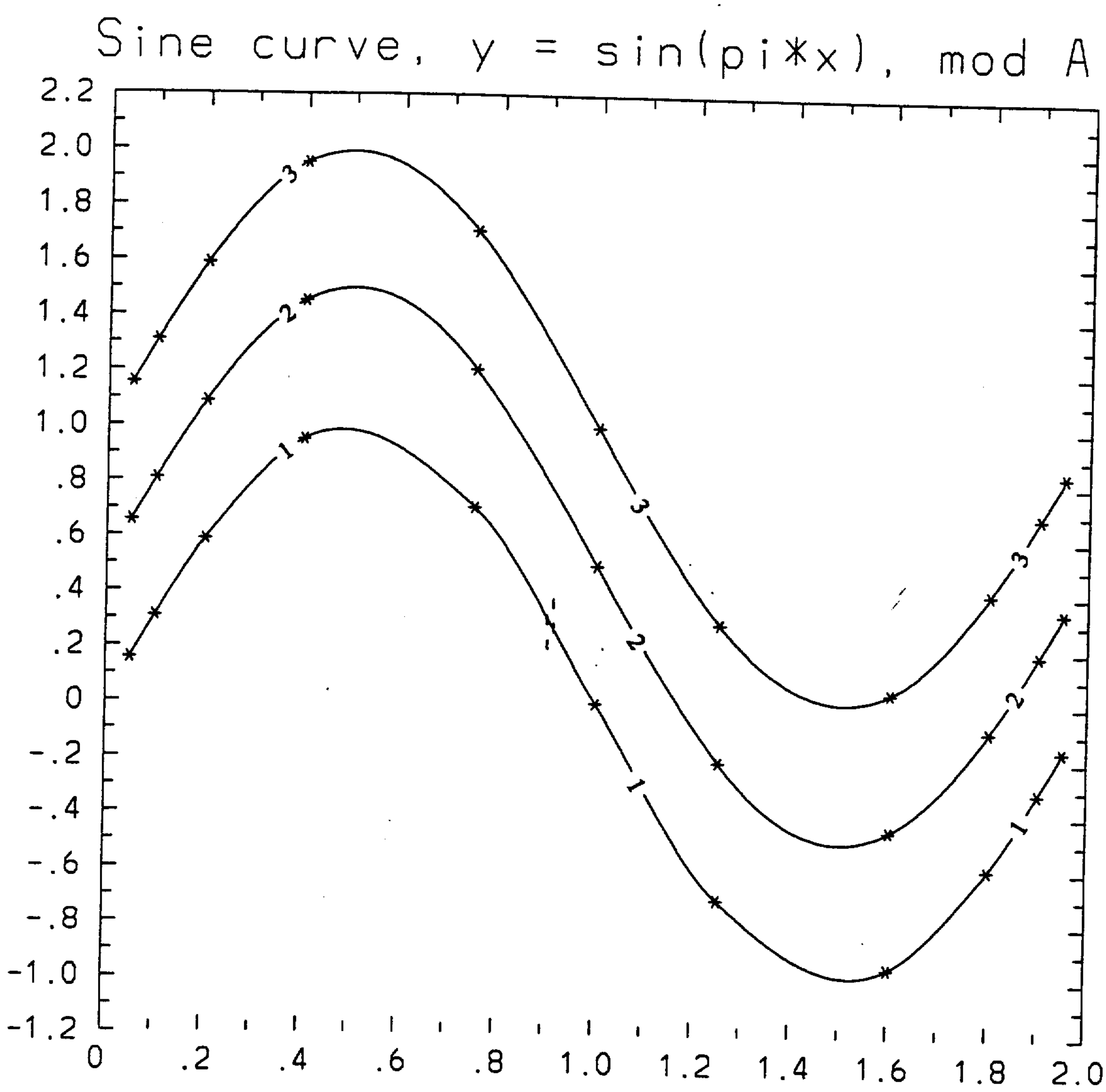
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 5



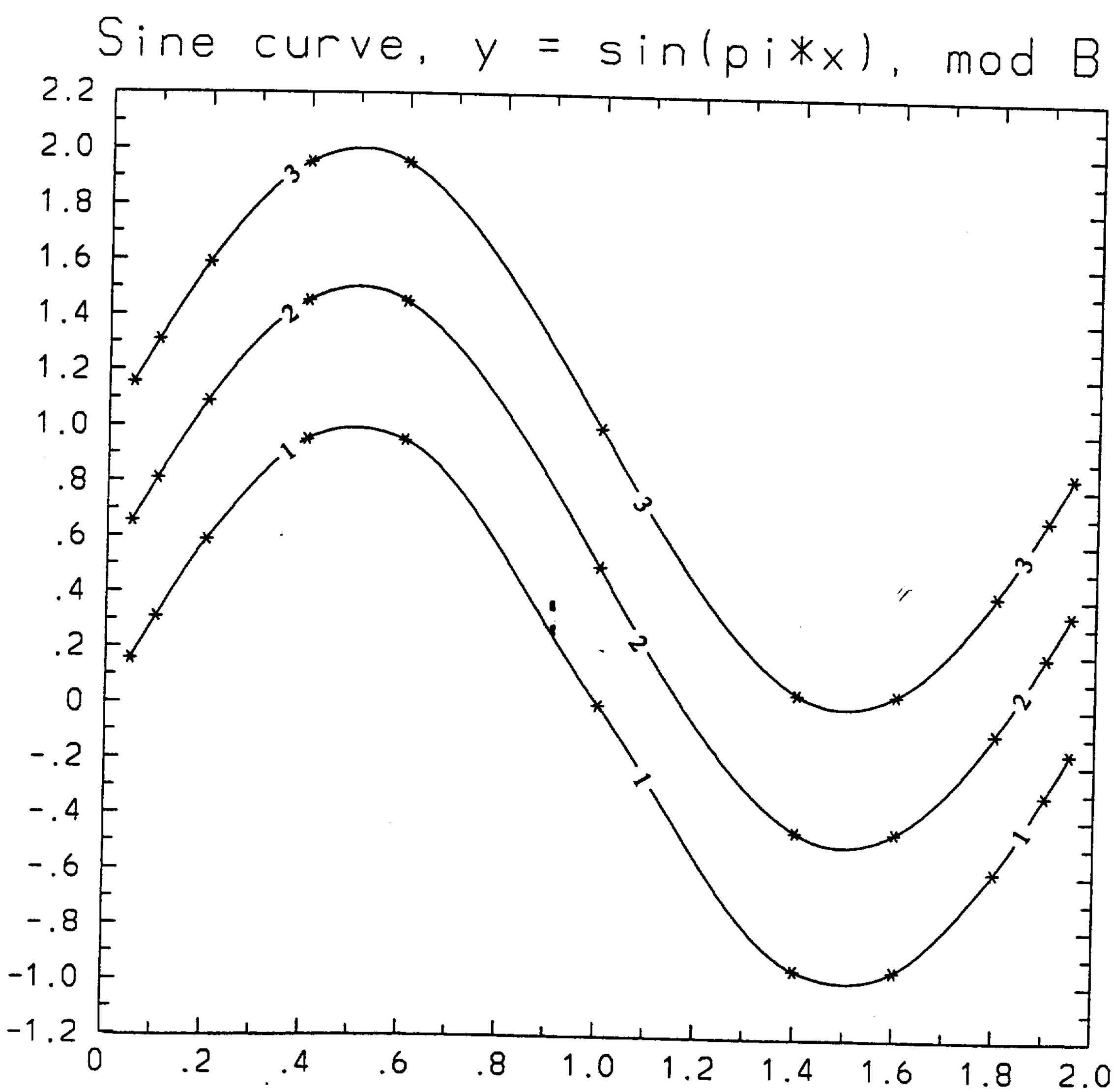
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 5a



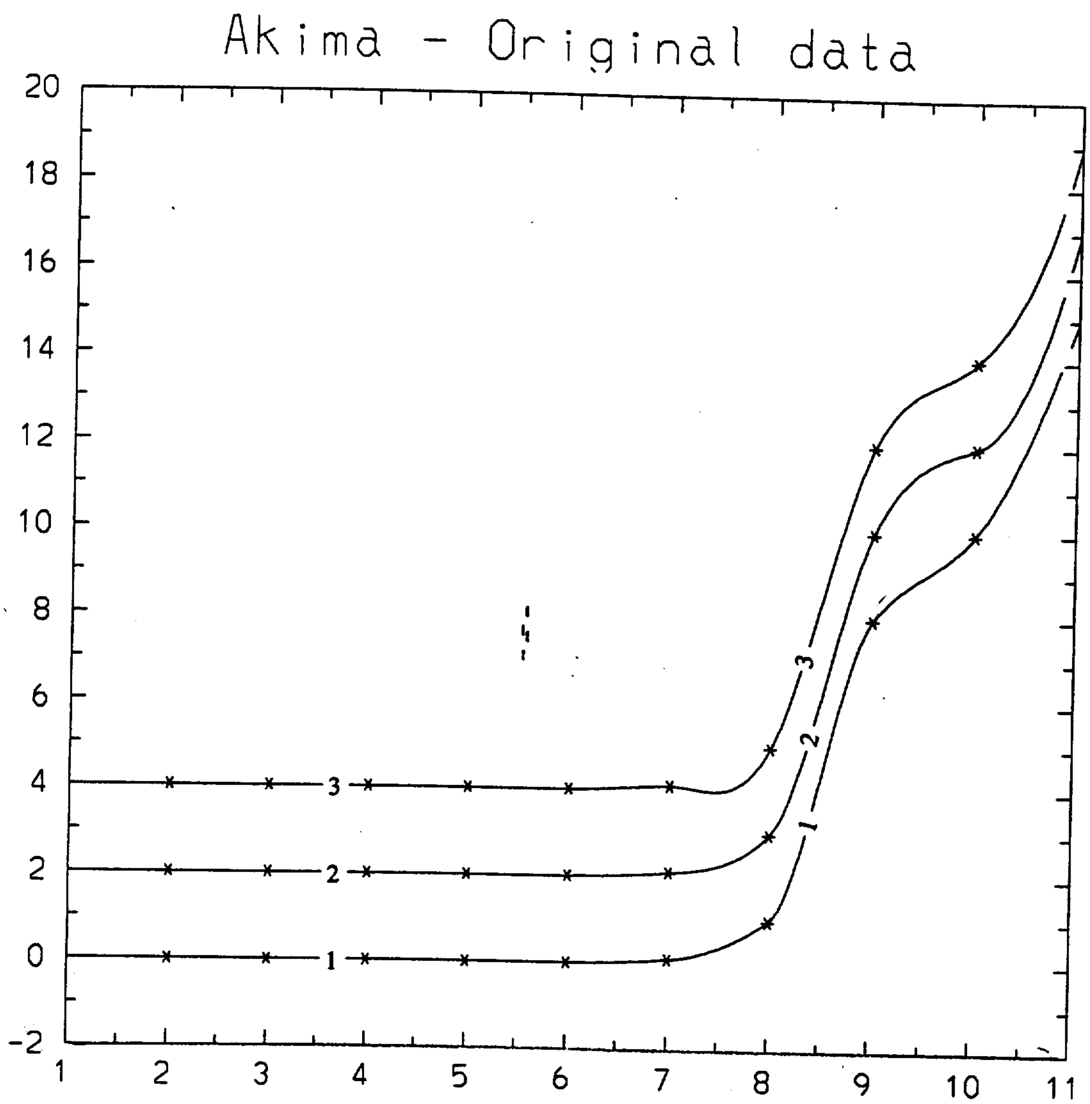
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 5b



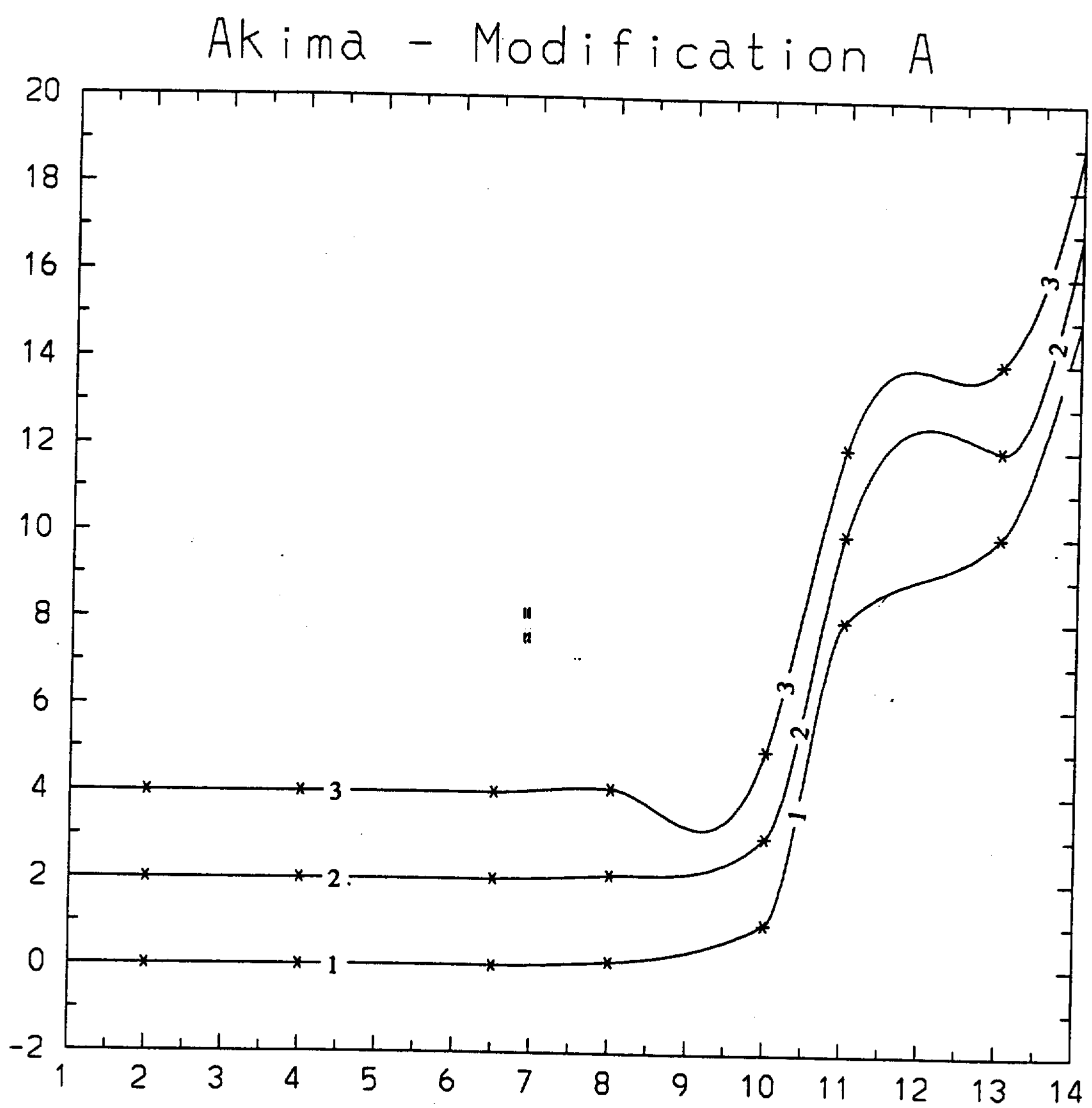
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 6



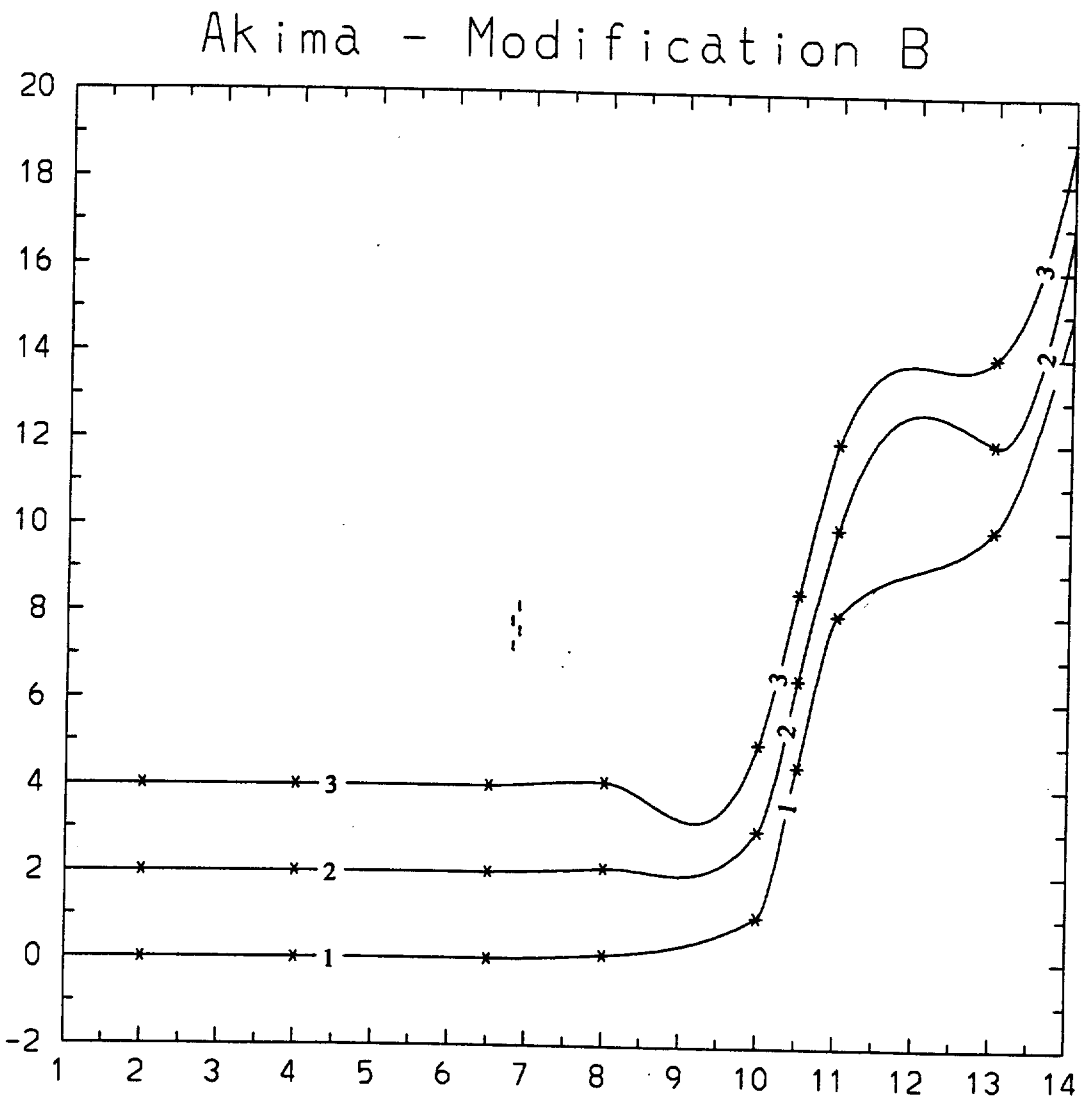
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 7



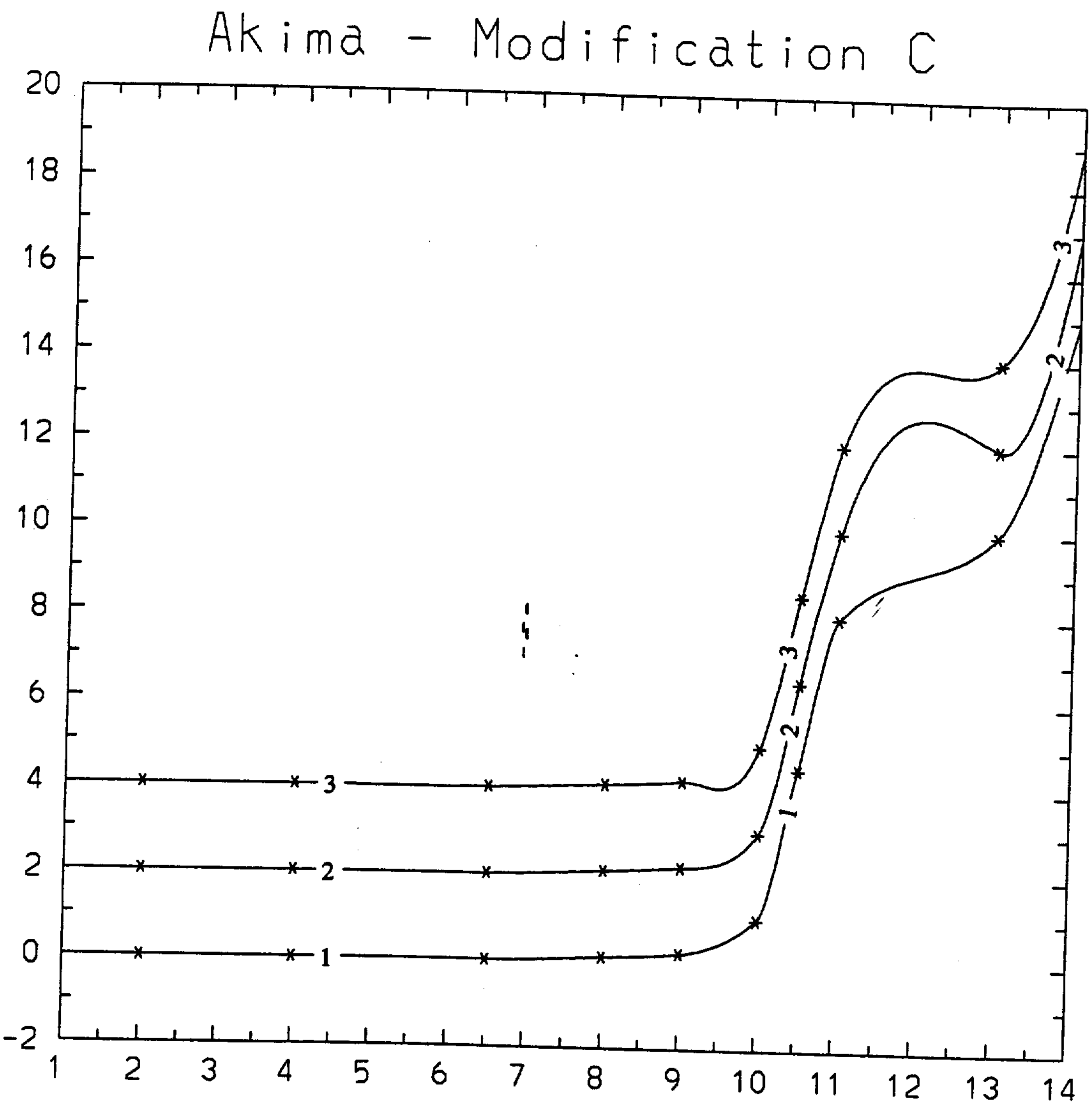
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 8



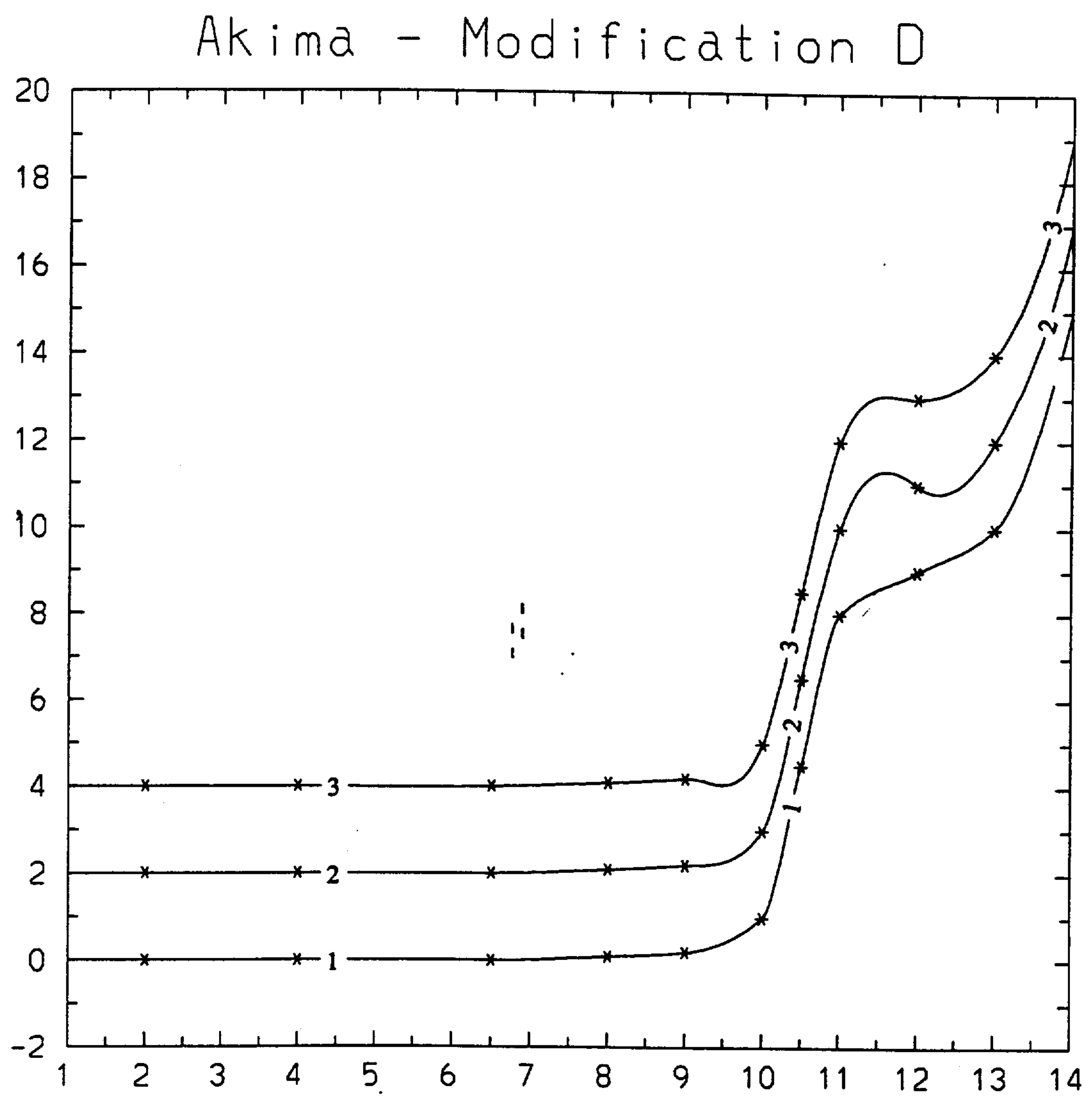
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 9



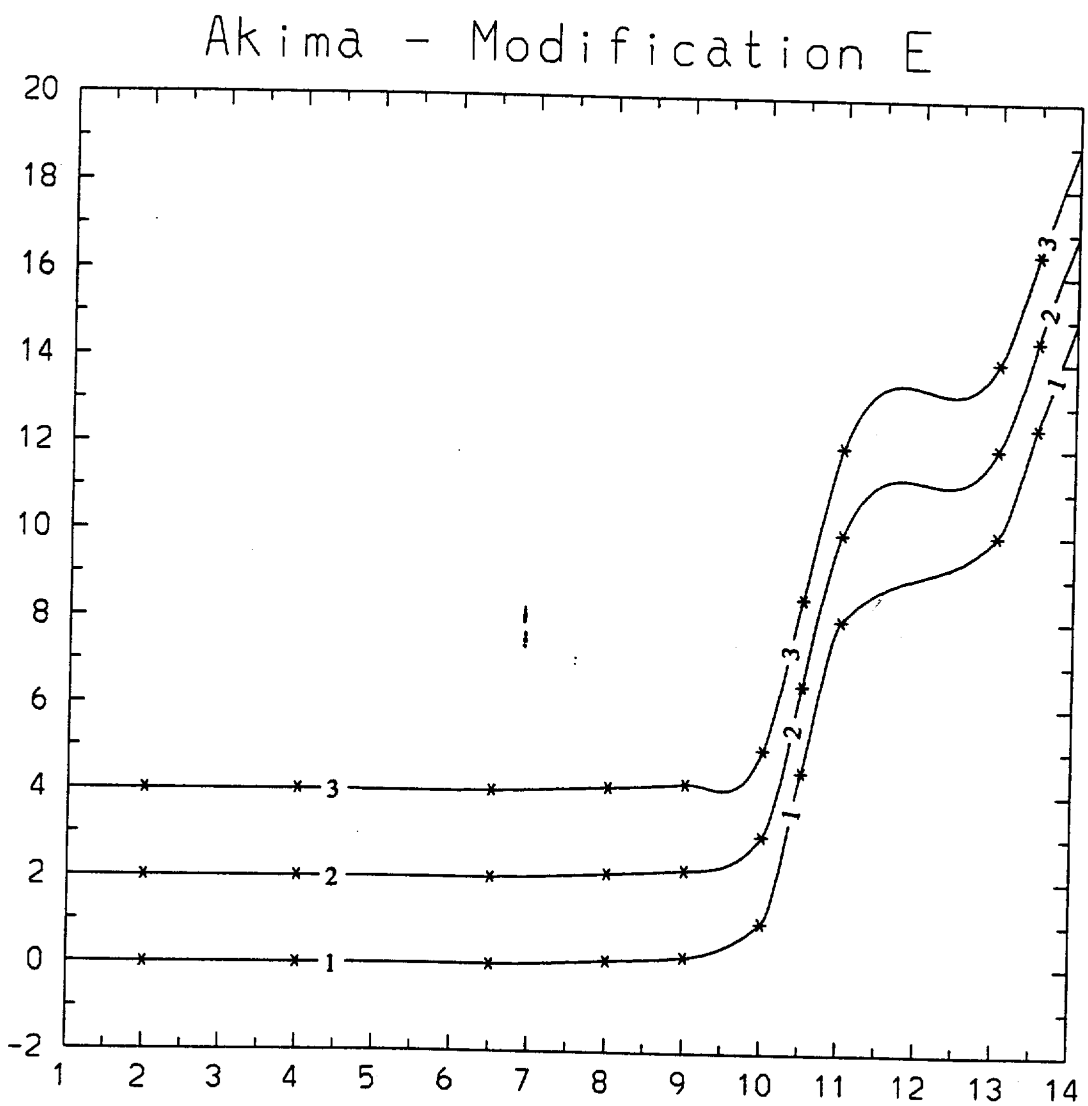
1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 10



1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514

Fig. 11



1: PCHIC (default B.C., SWITCH=-1)
2: Algorithm 697
3: Algorithm 514